

MATH 2 - JUNE 2005 - 1

Roll No.

Total No. of Pages : 3

BT-2/M05

8217

Mathematics-II (2003-04)

Paper : MATH-102E

Time : Three Hours]

[Maximum Marks : 100

Note :- Attempt FIVE questions in all, selecting at least ONE question from each part.

PART-A

1. (a) For the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

find non-singular matrices P and Q such that PAQ is in the normal form. Hence find the rank of A. 10

- (b) When is a square matrix said to be orthogonal? Verify that the matrix

$$\begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

is orthogonal. 10

2. (a) Find the values of a and b for which the equations $x + ay + z = 3$, $x + 2y + 2z = b$, $x + 5y + 3z = 9$ are consistent. When will these equations have a unique solution? 10
- (b) Find the characteristic roots and the corresponding characteristic vectors of the matrix 10

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

8217(01)

Contd.

MATH 2 - JUNE 2005 - 2

PART-B

3. (a) Solve the following equations :

(i) $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$

(ii) $(x^2 + y^2 - a^2)x dx + (x^2 - y^2 - b^2)y dy = 0$ 5+5

- (b) Find the orthogonal trajectory of the cardioids

$r = a(1 - \cos \theta)$. 10

4. (a) Solve the differential equation

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x e^x \sin x$. 10

- (b) Solve, by the method of variations of parameters,

$y'' - 2y' + y = e^x \log x$ 10

5. (a) Solve

$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$. 10

- (b) Solve the simultaneous equations

$\frac{dx}{dt} + 5x - 2y = t$; $\frac{dy}{dt} + 2x + y = 0$

given that $x = y = 0$ when $t = 0$ 10

PART-C

6. (a) Find the linear transform of

(i) $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$; (ii) $te^{-t} \sin 3t$ 5+5

- (b) Find the inverse Laplace forms of

(i) $\frac{s}{(s^2 - 1)^2}$; (ii) $\frac{4s + 5}{(s - 1)^2(s + 2)}$ 5+5

8217(02)

2

Contd.

MATH 2 - JUNE 2005 - 3

7. (a) Solve the equation using Laplace transform
 $y'' + 4y' + 3y = e^{-t}$, where $y(0) = y'(0) = 1$ 10

(b) (i) Find the Laplace transform of the function

$$f(t) = \begin{cases} \sin wt, & 0 < t < \pi/w; \\ 0, & \pi/w < t < 2\pi/w \end{cases}$$

(ii) Find the inverse Laplace transform of

$$\frac{e^{-cs}}{s^2(s+a)} \quad (c > 0)$$

5+5

8. (a) Form the partial differential equation by eliminating the arbitrary constants from

$$z = a(x+y) + b(x-y) + abt + c.$$

8

(b) Solve the partial differential equation

$$x^2 p^2 + y^2 q^2 = z^2.$$

6

(c) Solve the partial differential equation using Charpit's method.

$$2z + p^2 + qy + 2yz = 0$$

6